Magnetotransport in three-dimensional systems placed in quantizing magnetic field

M. V. Cheremisin
A.F.Ioffe Physical-Technical Institute, St.Petersburg, Russia

Based on a thermodynamic approach, we calculate the components of the resistivity tensor for 3D electron gas assumed to be dissipationless in the strong quantum limit. The longitudinal resistivity results from Peltier and Seebeck effects combined. The components of the resistivity tensor found to be a universal functions of magnetic field and temperature.

PACS numbers: 73.50.Jt, 73.40.Hm, 73.61.Ey

The electronic transport in 3D solids subjected in strong magnetic fields have been of intense investigation since the late 1950s. Usually, the transport coefficients can be easily found out by means of standard calculation of the bulk current and heat fluxes. As was demonstrated first in Ref. [1], in quantizing magnetic fields it is necessary to take into account the diamagnetic surface currents. The total current through the sample cross section is given by the sum of both bulk and surface components. In this case the transport coefficients found to satisfy the Einstein and Onsager phenomenological relationships.

The main goal of this paper is to present a thermodynamic approach to the problem of the 3D electron transport in strong quantum limit. In this case, the thermodynamics of reversible processes known to be valid. [1] We further consider dissipationless 3D electron gas when both the conductivity and resistivity tensors are purely off-diagonal. We calculate these components using Gibbs statistics for variable number of particles assuming that the chemical potential of 3D electrons is pinned by an external carriers reservoir. Indeed, according to conventional thermodynamics [2] the chemical potential, μ, for the system conductors+sample is constant at the thermodynamic equilibrium. Thus, we conclude that 3D electron gas is not, in fact, isolated. We argue that the metal leads can play the role of such an electron reservoir. Let us suppose first that the chemical potential of 3D electron gas is fixed. Evidently, in this case the concentration, N, of 3D electrons changes with magnetic field. In a strong magnetic field, when only few LLs are filled, the absolute change in N is of the order of the 3D electron concentration, N_0, at $B = 0$. Accordingly, the number of external reservoir electrons (metal leads) changes. The relative variation of the chemical potential of the reservoir is given by $\delta\mu/\mu = \frac{2}{3}N_0/N_m$, where $N_m \approx 10^{23}\mathrm{cm}^{-3}$ is the concentration of the electrons in the metal. For a typical InSb sample with $N_0 = 10^{16}\mathrm{cm}^{-3}$ we have $\delta\mu/\mu \approx 10^{-7}$. We conclude that the chemical potential of the 3D system is always fixed under the above conditions. Moreover, we support this idea by calculating Debye screening length for 3D electron plasma in quantizing magnetic field. It will be shown that the screening length is of the order of macroscopic length scale, thus the conventional arguments concerning the electron plasma neutrality in the sample bulk are violated. Thus, the applicability of Gibbs approach for 3D electron gas is justified.

Let us envisage an isotropic and homogeneous conductor subjected to a perpendicular magnetic field $B = B_z$. Without spin splitting, the 3D energy spectrum is $\varepsilon_n = \hbar\omega_c(n + 1/2) + \varepsilon_{\shortparallel}$, where $n = 0, 1...$ is the Landau level (LL) number, $\omega_c = eB/mc$ is the cyclotron frequency, m is the electron effective mass. Then, $\varepsilon_{\shortparallel} = \frac{p_z^2}{2m}$ is the kinetic energy of an electron moving parallel to the magnetic field. Without broadening the LL-related density of states is given by $\Gamma = 1/2\pi l_B^2$, where $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length. In present paper we disregard valley degeneracy, and, then restrict ourselves within strong quantum limit when $\hbar\omega_c >> kT$.

We now consider a standard experimental setup with Hall-bar geometry sample (Fig.1, insert). The sample is connected by means of two identical leads to the current source. Both the contacts ("a" and "b") assumed to be ohmic. The voltage is measured between the open ends ("c" and "d") kept at the temperature of the external thermal reservoir. The sample is placed in a sample chamber (not shown) with mean temperature T_0. In general, the macroscopic current, $\mathbf{j}$, and the energy flux, $\mathbf{q}$, densities are given by

$$\begin{aligned} \mathbf{j} &= \hat{\sigma}(\mathbf{E} - \alpha\nabla T), \\ \mathbf{q} &= (\alpha T - \zeta/e)\,\mathbf{j} - \hat{\varkappa}\nabla T. \end{aligned} \tag{1}$$

Here, $\mathbf{E} = \nabla\zeta/e$ is the electric field, $\zeta = \mu - e\varphi$ is the electrochemical potential, and α is the thermopower. Then, $\hat{\sigma}$, $\hat{\varkappa} = LT\hat{\sigma}$ are the conductivity and electron-related thermal conductivity tensors respectively, and $L = \frac{\pi^2 k^2}{3e^2}$ is the Lorentz number. It will be recalled that Eq.(1) is valid for a confined-topology sample for which the diamagnetic surface currents [1] are accounted. Both Einstein and Onsager relationships are satisfied. Actually, within strong quantum limit Eq.(1) represents x-y plane current and energy dissipationless flux caused by electron drift in crossed fields [3]. The x-y plane components of the conductivity $\hat{\sigma}$and resistivity, $\hat{\rho} = (\hat{\sigma})^{-1}$, tensors are purely off-diagonal. Then, the z-component of the conductivity tensor is given by the conventional Drude formulae $\sigma_{zz} = Ne^2\tau/m$, where N is the electron density. For simplicity, we further assume that τ is the energy independent momentum relaxation time.

Let us first examine the charge relaxation and screening for 3D solids placed in quantizing magnetic field. In general, this effect is described by the continuity equation

$$\frac{\partial \rho_e}{\partial t} + \mathrm{div}\mathbf{j} = 0, \qquad (2)$$

where $\rho_e = -Ne$ is the electron charge density. Let us suppose, for example, a thin charged thread (|| z) with fixed charge density, q, per unit length. In this case, the problem becomes independent on z. Using Eq.(1,2) for isotropic conductor($\sigma_{ii} = \sigma$) at $B = 0$ and $\nabla T = 0$ we obtain the conventional screening as $\rho_e = -\frac{q}{2\pi} K_0(r/l)$. Here, K_0 is the first order modified Bessel function of the second kind, $l = (D\tau_M)^{1/2}$ is the Debye screening length, $D = \frac{\sigma}{e^2}\left(\frac{dN}{d\mu}\right)^{-1}$ is the diffusion coefficient, $\tau_M = (4\pi\sigma)^{-1}$ is the Maxwell relaxation time. In strong magnetic field the screening scenario, however, essentially differs from that described above. Neglecting retardation effects, the current given by Eq.(1) can be re-written [3] as follows $\mathbf{j} = -\frac{cN}{B^2}\mathrm{rot}(\zeta\mathbf{B})$. At fixed magnetic field we obtain $\mathrm{div}\mathbf{j} = 0$, therefore, $\partial\rho_e/\partial t = 0$. The retardation effects, if accounted, result in slow charge relaxation analogous to 2DEG case [4] when the amount of charge spread in x-y plane found to be proportional to $(\sigma_{yx}/c)^2$. We argue that Debye screening length could be of the macroscopic length scale for 3D electrons places in quantizing magnetic fields. Thus, the electron plasma neutrality can be violated in the 3D sample bulk.

We now discuss the real measurements of the longitudinal resistivity. It will be remind that for dissipationless approach in question $\sigma_{xx} = \rho_{xx} = 0$. Nevertheless, the extraneous resistivity can arise from Peltier and Seebeck thermoelectric effects combined. [5]- [7] It is well known that the Peltier heat is generated by a current crossing the contact of two different conductors. At the contact (for example "a" in Fig.1, insert) the temperature, T_a, electrochemical potential ζ, normal components of the total current, I, and the total energy flux are continuous. There exists a difference $\Delta\alpha = \alpha_m - \alpha$ between the thermopowers of the metal conductor and 3D sample, respectively. Then, $Q_a = I\Delta\alpha T_a$ is the amount of Peltier heat evolved per unit time in the contact "a". For $\Delta\alpha > 0$ and current flow direction shown in Fig.1, the contact "a" is heated and the contact "b" is cooled. The contacts are at different temperatures, and $\Delta T = T_a - T_b > 0$. At small currents, the temperature gradient is small and $T_{a,b} \approx T_0$. In this case, α can be assumed to be constant, and, hence, we disregard the Thomson heating($\sim IT\nabla\alpha$) of 3D electrons. We argue that within low temperature measurements the electron temperature could be greater than the bath temperature (for 2D case, see Ref. [8]). The electron gas cooling could occur predominantly through the contacts of the sample on the leads connected to them because of weak electron-phonon coupling. However, the heat leak via the metal leads can be also neglected similar to that in 2DEG case. [8] Finally, we will consider 3D sample under adiabatic cooling conditions.

One can easily demonstrate that the amount of the Peltier heat evolved at the contact "a" is equal to that absorbed at the contact "b." It will be recalled that the current is known to enter and leave the sample at two diagonally opposite corners (Fig.1, insert). Since the energy flux is continuous at each contact, we finally obtain $\nabla_x T = -j\Delta\alpha T/\varkappa_{yx}$. As expected, the temperature gradient is linear in current. Then, the transverse electron flow associated with the above temperature gradient is compensated by that due to the longitudinal electric field, since $j_y = 0$. Neglecting ohmic resistance of the metal leads the total voltage measured between the open ends ("c" and "d") is equal to Seebeck thermoemf $U = \int_c^d \alpha dT = \Delta\alpha\Delta T$. The associated resistivity is given $\rho = U/jl$. Finally, the components of the resistivity tensor $\hat{\rho} = (\hat{\sigma})^{-1}$ yield

$$\rho_{yx} = \frac{B}{Nec}, \qquad \rho = s\rho_{yx}, \qquad \rho_{zz} = \frac{\rho_{yx}}{\omega_c\tau}, \qquad (3)$$

where $N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,B}$ is the 3D electron density, $\Omega = -2kT\cdot\Gamma\sum_{n,p_z}\ln(1+\exp((\mu-\varepsilon_n)/kT))$ is the thermodynamic potential, $s = \alpha^2/L$ is the dimensionless parameter. In Eq.(3) we take into account that for the actual case of metal leads $\Delta\alpha \simeq -\alpha$. According to Eq.(3), the power dissipated in the sample is always positive since $\rho j^2 > 0$. It will be recalled that in strong quantum limit, the thermopower of dissipationless 3D electron gas is a universal thermodynamic quantity proportional to the entropy per one electron: [1]

$$\alpha = -\frac{S}{eN}, \qquad (4)$$

where $S = -\left(\frac{\partial\Omega}{\partial T}\right)_{\mu,B}$ is the 3D electron entropy. One can demonstrate that both the electron density N and entropy S are the universal functions of the reduced temperature $\xi = kT/\mu$ and dimensionless magnetic field $\hbar\omega_c/\mu = \nu^{-1}$, where ν is the so-called filling factor. Following to conventional Lifshitz-Kosevich formalism [9] we can easily derive asymptotic formulae for N, S, and, hence, for α, valid within the low temperature and low magnetic field limit $\nu^{-1}, \xi < 1$:

$$N = N_0\left(1 + \frac{\pi^2}{2F_{1/2}(1/\xi)}\sum_{k=1}^{\infty}\frac{(-1)^k\sin(2\pi k\nu - \pi/4)}{\sqrt{z}\sinh z}\right), \qquad (5)$$

$$S = S_0 + N_0 k\frac{\pi^3}{2F_{1/2}(1/\xi)}\sum_{k=1}^{\infty}(-1)^k\Phi(z)\cos(2\pi k\nu - \pi/4),$$

where $\Phi(z) = \frac{z\coth z - 1}{z^{3/2}\cdot\sinh z}$, $z = 2\pi^2\xi\nu k \sim kT/\hbar\omega_c$ is the dimensionless parameter. Then, $N_0 = n_0\frac{3}{2}\xi^{3/2}F_{1/2}(1/\xi)$ and $S_0 = N_0 k\left(\frac{5F_{3/2}(1/\xi)}{3F_{1/2}(1/\xi)} - \frac{1}{\xi}\right)$ are the 3D electron gas concentration and entropy at $B = 0$ respectively, $F_n(y)$ is the Fermi integral, $n_0 = \frac{(2m\mu)^{3/2}}{3\pi^2\hbar^3}$ is the density of the strongly degenerated electron gas at $T = 0$.

In Fig.(1) we plot the dependences $\rho_{yx}(\nu^{-1})$, $\rho(\nu^{-1})$ given by Eqs.(3,4). Both dependences are scaled in units

$\rho_c = \frac{m\mu}{n_0 e^2 \hbar} = \frac{\hbar}{e^2}\frac{3\pi}{4k_F}$, where $k_F = \sqrt{2m\mu}/\hbar$ is the Fermi vector. Then, the dependence $\rho_{zz}(\nu^{-1})$ is scaled in units of zero magnetic field resistivity $\rho_{zz}(0) = \frac{m}{n_0 e^2 \tau} = \frac{2\rho_c}{k_F l}$, where $l = \hbar k_F \tau/m$ is the mean free path. The magnetic field dependence of the resistivity tensor components can be easily understood in terms of 3D density of states. Really, in addition to discrete set of LLs the 3D energy spectrum contains a component associated with the electron motion along the magnetic field. At fixed μ the number of occupied states, and, hence ρ_{yx}, ρ_{zz} are smooth functions of the magnetic field, as well as LLs cross the Fermi energy level. Then, the resistivity ρ remains nonzero because of continuous density of states. In contrast to 3D electron gas, in 2D case the energy spectrum is essentially discrete, therefore leads to quantized Hall resistivity ρ_{yx}, while the longitudinal resistivity ρ vanishes within Hall plateaux.

We now compare our results with experiments. [10] At helium temperatures for typical n-InSb sample ($n_0 = 1.2{\cdot}10^{16}\mathrm{cm}^{-3}$, $\mu_e = 1.3{\cdot}10^5\mathrm{cm}^2/\mathrm{Vs}$) we obtain $\mu = 140\mathrm{K}$, $\xi = 0.03$ and, therefore $\rho_c = 0.08\mathrm{Om{\cdot}cm}$. Thus, the Peltier effect related resistivity is $\rho \simeq \rho_c/20 = 0.004$ Om·cm and hence, is consistent with the values ~ 0.01 Om·cm found experimentally.

In conclusion, we found out the magnetoresistivity tensor components for 3D electrons placed in strong quantizing field. The transport coefficients shown are universal function of magnetic field and temperature.

This work was supported by RFBR, INTAS (grant YSF 2001/1-0132), and the LSF program (Weizmann Institute).

FIG. 1. The dimensionless resistivities ρ_{yx}, ρ (scaled by ρ_c) and $\rho_{zz}/\rho_{zz}(0)$ vs ν^{-1} for $\xi = 0.025$. The dashed(dotted) line represents the classical result for ρ_{yx} and ρ_{zz} respectively. Inserts: (left) low magnetic field dependence $\rho(\nu^{-1})$ given by Eq.(3,5); (right) Hall bar experimental setup.

[1] Yu.N.Obraztsov, Sov. Phys. Solid State, **6** , 331,1964, *ibid* **7**, 455, 1965

[2] L.D.Landau, E.M.Lifshits, Statistical Physics, Pergamon, New York, 1966

[3] P.S.Zyryanov, Fizika Tverdogo Tela, **6**, 3562, 1964

[4] A.O.Govorov, A.V.Chaplik, Zh. Eksp.Teor.Fiz., **95**, 1976, 1989

[5] C.G.M.Kirby, M.J.Laubitz, Metrologia, **9**, 103, 1973

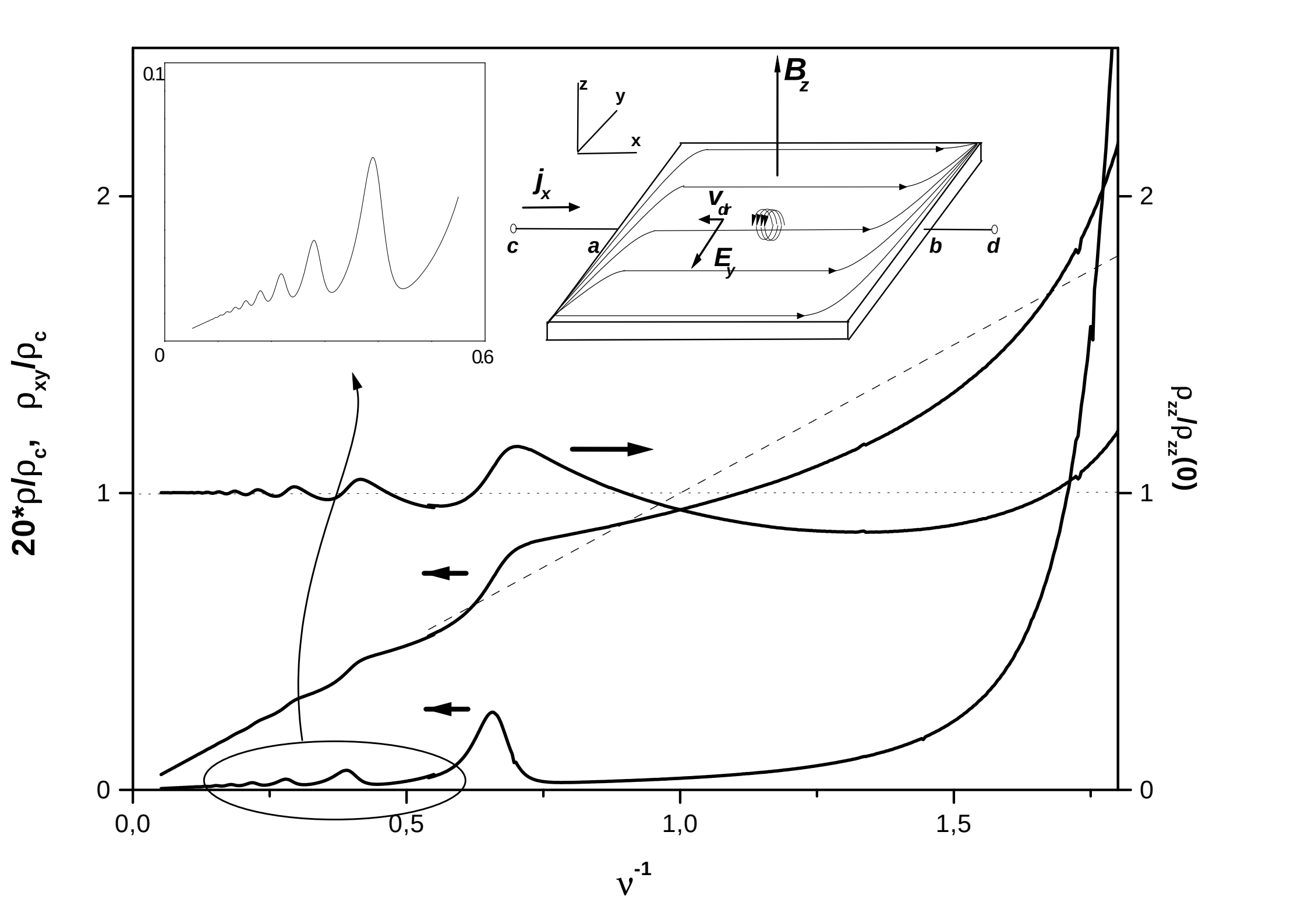
[6] M.V.Cheremisin, Sov. Phys. JETP, **92**, 357, 2001

[7] M.V.Cheremisin, Proceedings of NATO ASI, Windsor, UK, 2001, Phys.Rev.B., in progress

[8] A.Mittal et al., Physica B, **194-196**, 167, 1994

[9] I.M.Lifshitz, A.M.Kosevich, Sov.Phys.JETP, **29**, 730, 1955

[10] F.A.Egorov, S.S.Murzin, Zh. Eksp. Teor. Fiz., **94**, 3165, 1988


0.1
0
0.6
B_z
z
y
x
j_x
v_d
E_y
c
a
b
d
2
1
0
20*ρ/ρ_c, ρ_xy/ρ_c
ρ_zz/ρ_zz(0)
0,0
0,5
1,0
1,5
ν^-1